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Friendship networks and social status

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In empirical studies of friendship networks participants are typically asked, in interviews or questionnaires, to identify some or all of their close friends, resulting in a directed network in which friendships can, and often do, run in only one direction between a pair of individuals. Here we analyze a large collection of such networks representing friendships among students at US high and junior-high schools and show that the pattern of unreciprocated friendships is far from random. In every network, without exception, we find that there exists a ranking of participants, from low to high, such that almost all unreciprocated friendships consist of a lower-ranked individual claiming friendship with a higher-ranked one. We present a maximum-likelihood method for deducing such rankings from observed network data and conjecture that the rankings produced reflect a measure of social status. We note in particular that reciprocated and unreciprocated friendships obey different statistics, suggesting different formation processes, and that rankings are correlated with other characteristics of the participants that are traditionally associated with status, such as age and overall popularity as measured by total number of friends.

Introduction

A social network, in the most general sense of the term, consists of a group of people, variously referred to as nodes or actors, connected by social interactions or ties of some kind [1]. In this paper we consider networks in which the ties represent friendship. Friendship networks have been the subject of scientific study since at least the 1930s. A classic example can be found in the studies by Rapoport and collaborators of friendship among schoolchildren in the town of Ann Arbor, MI in the 1950s and 60s [2], in which the investigators circulated questionnaires among the students in a school asking them to name their friends. Many similar studies have been done since then, with varying degrees of sophistication, but most employ a similar questionnaire-based methodology. A counterintuitive aspect of the resulting networks is that they are directed. Person A states that person B is their friend and hence there is a direction to the ties between individuals. It may also be that person B states that person A is their friend, but it does not have to be the case, and in practice it turns out that a remarkably high fraction of claimed friendships are not reciprocated. In the networks we study in this paper the fraction of reciprocated ties rarely exceeds 50% and can be as low as 30%.

This could be seen as a problem for the experimenter. One thinks of friendship as a two-way street—a friendship that goes in only one direction is no friendship at all. How then are we to interpret the many unreciprocated connections in these networks? Are the individuals in question friends or are they not? One common approach is simply to disregard the directions altogether and consider two individuals to be friends if they are connected in either direction (or both) [3]. In this paper, however, we take a different view and consider what we can learn from the unreciprocated connections. It has been conjectured that, rather than being an error or an annoyance, the pattern of connections might reflect underlying features in the structure or dynamics of the community under study [4–6].

Working with a large collection of friendship networks from US schools, we find that in every network there is a clear ranking of individuals from low to high such that almost all friendships that run in only one direction consist of a lower-ranked individual claiming friendship with a higher-ranked one. We conjecture that these rankings reflect a measure of social status and present a number of results in support of this idea. For instance, we find that a large majority of reciprocated friendships are between individuals of closely similar rank, while a significant fraction of unreciprocated friendships are between very different ranks, an observation consistent with qualitative results in the sociological literature going back several decades [5]. We also investigate correlations between rank and other individual characteristics, finding, for example, that there is a strong positive correlation between rank and age, older students having higher rank on average, and between rank and overall popularity, as measured by total number of friends.

The outline of the paper is as follows. First, we describe our method of analysis, which uses a maximum-likelihood technique in combination with an expectation–maximization algorithm to extract rankings from directed network data. Then we apply this method to school friendship networks, revealing a surprisingly universal pattern of connections between individuals in different schools. We also present results showing how rank correlates with other measures. Finally we give our conclusions and discuss possible avenues for future research.

Inference of rank from network structure

Consider a directed network of friendships between n individuals in which a connection running from person A to person B indicates that A claims B as a friend. Suppose that, while some of the friendships in the network

may be reciprocated or bidirectional, a significant fraction are unreciprocated, running in one direction only, and suppose we believe there to be a ranking of the individuals implied by the pattern of the unreciprocated friendships so that most such friendships run from lower to higher rank. One possible way to infer that ranking would be simply to ignore any reciprocated friendships and then construct a minimum violations ranking of the remaining network [7, 8]. That is, we find the ranking of the network nodes that minimizes the number of connections running from higher ranked nodes to lower ranked ones. In practice this approach works quite well: for the networks studied in this paper the minimum violations rankings have an average of 98% of their unreciprocated friendships running from lower to higher ranks and only 2% running the other way. By contrast, versions of the same networks in which edge directions have been randomized typically have about 10% of edges running the wrong way. (Statistical errors in either case are 1% or less, so these observations are highly unlikely to be the results of chance.)

The minimum violations ranking, however, misses important network features because it focuses only on unreciprocated friendships. In most cases there are a substantial number of reciprocated friendships as well, as many as a half of the total, and they contain significant information about network structure and ranking. For example, as we will see, pairs of individuals who report a reciprocated friendship are almost always closely similar in rank. To make use of this information we need a more flexible and general method for associating rankings with network structure. In this paper we use a maximum likelihood approach defined as follows.

Mathematically we represent the distinction between reciprocated and unreciprocated friendships in the network using two separate matrices. The symmetric matrix $\mathbf{S}$ will represent the reciprocated connections—undirected edges in graph theory terms—such that $S_{ij} = S_{ji} = 1$ if there are connections both ways between nodes i and j, and zero otherwise. The asymmetric matrix $\mathbf{T}$ will represent the unreciprocated (directed) edges with $T_{ij} = 1$ if there is a connection to node i from node j (but not *vice versa*), and zero otherwise. The matrices $\mathbf{S}$ and $\mathbf{T}$ are related to the conventional adjacency matrix $\mathbf{A}$ of the network by $\mathbf{A} = \mathbf{S} + \mathbf{T}$.

Now suppose that there exists some ranking of the individuals, from low to high, which we will represent by giving each individual a unique integer rank in the range 1 to n. We will denote the rank of node i by r_i and the complete set of ranks by R. We have found it to be a good approximation to assume that the probability of friendship between two individuals is a function only of the difference between their ranks. We specifically allow the probability to be different for reciprocated and unreciprocated friendships, which acknowledges the possibility that the two may represent different types of relationships, as conjectured for instance in [5, 9]. We define a function $\alpha(r_i - r_j)$ to represent the probability of an undirected edge between i and j and another $\beta(r_i - r_j)$ for a directed edge to i from j. Since $\alpha(r)$ describes undirected edges it must be symmetric $\alpha(-r) = \alpha(r)$, but $\beta(r)$ need not be symmetric.

If we were not given a network but we were given the probability functions α and β and a complete set of rankings on n vertices, then we could use this model to generate—for instance on a computer—a hypothetical but plausible network in which edges appeared with the appropriate probabilities. In effect, we have a random graph model that incorporates rankings. In this paper, however, we want to perform the reverse operation: given a network we want to deduce the rankings of the nodes and the values of the functions α and β. To put that another way, if we are given a network and we assume that it is generated from our model, what values of the rankings and probability functions are most likely to have generated the network we observe?

This question leads us to a maximum likelihood formulation of our problem, which we treat using an expectation–maximization (EM) approach in which the ranks R are considered hidden variables to be determined and the functions α and β are parameters of the model. Using a Poisson formulation of the random network generation process, we can write the probability of generation of a network G with rankings R, given the functions α and β, as

$$P(G,R|\alpha,\beta) = \prod_{i>j} \frac{[\alpha(r_i - r_j)]^{S_{ij}}}{S_{ij}!}\,\mathrm{e}^{-\alpha(r_i-r_j)} \times \prod_{i\neq j} \frac{[\beta(r_i - r_j)]^{T_{ij}}}{T_{ij}!}\,\mathrm{e}^{-\beta(r_i-r_j)}. \tag{1}$$

Note that we have excluded self-edges here, since individuals cannot name themselves as friends. We have also assumed that the prior probability of R is uniform over all sets of rankings.

The most likely values of the parameter functions α and β are now given by maximizing the marginal likelihood $P(G|\alpha,\beta) = \sum_R P(G,R|\alpha,\beta)$, or equivalently maximizing its logarithm, which is more convenient. The logarithm satisfies the Jensen inequality

$$\log \sum_R P(G,R|\alpha,\beta) \geq \sum_R q(R) \log \frac{P(G,R|\alpha,\beta)}{q(R)}, \tag{2}$$

for any set of probabilities $q(R)$ such that $\sum_R q(R) = 1$, with the equality being recovered when

$$q(R) = \frac{P(G,R|\alpha,\beta)}{\sum_R P(G,R|\alpha,\beta)}. \tag{3}$$

This implies that the maximization of the log-likelihood on the left side of (2) is equivalent to the double maximization of the right side, first with respect to $q(R)$, which makes the right side equal to the left, and then with respect to α and β, which gives us the answer we

are looking for. It may appear that expressing the problem as a double maximization in this way, rather than as the original single one, makes it harder, but in fact that's not the case.

The right-hand side of (2) can be written as $\sum_R q(R) \log P(G,R|\alpha,\beta) - \sum_R q(R)\log q(R)$, but the second term does not depend on α or β, so as far as α and β are concerned we need consider only the first term, which is simply the average $\overline{\mathscr{L}}$ of the log-likelihood over the distribution $q(R)$:

$$\overline{\mathscr{L}} = \sum_R q(R) \log P(G,R|\alpha,\beta). \tag{4}$$

Making use of Eq. (1) and neglecting an unimportant overall constant, we then have

$$\overline{\mathscr{L}} = \sum_R q(R) \sum_{i\neq j} \Bigl[\tfrac12 S_{ij}\log\alpha(r_i-r_j) + T_{ij}\log\beta(r_i-r_j) - \tfrac12\alpha(r_i-r_j) - \beta(r_i-r_j)\Bigr], \tag{5}$$

where we have used the fact that $\alpha(r)$ is a symmetric function.

This expression can be simplified further. The first term in the sum is

$$\tfrac12 \sum_R q(R) \sum_{i\neq j} S_{ij}\log\alpha(r_i-r_j) = \tfrac12 \sum_z \sum_{i\neq j} S_{ij} q(r_i-r_j=z)\log\alpha(z), \tag{6}$$

where $q(r_i - r_j = z)$ means the probability within the distribution $q(R)$ that $r_i - r_j = z$. We can define

$$a(z) = \frac{1}{n-|z|}\sum_{i\neq j} S_{ij} q(r_i-r_j=z), \tag{7}$$

which is the expected number of undirected edges in the observed network between pairs of nodes with rank difference z. It is the direct equivalent in the observed network of the quantity $\alpha(z)$, which is the expected number of edges in the model. The quantity $a(z)$, like $\alpha(z)$, is necessarily symmetric, $a(z) = a(-z)$, and hence (6) can be written as

$$\tfrac12 \sum_R q(R) \sum_{i\neq j} S_{ij}\log\alpha(r_i-r_j) = \sum_{z=1}^{n-1} (n-z) a(z)\log\alpha(z). \tag{8}$$

Similarly, we can define

$$b(z) = \frac{1}{n-|z|}\sum_{i\neq j} T_{ij} q(r_i-r_j=z) \tag{9}$$

and

$$\sum_R q(R)\sum_{i\neq j} T_{ij}\log\beta(r_i-r_j) = \sum_{z=1}^{n-1}(n-z)\bigl[b(z)\log\beta(z) + b(-z)\log\beta(-z)\bigr], \tag{10}$$

where $b(z)$ is the expected number of directed edges between a pair of nodes with rank difference z. Our final expression for $\overline{\mathscr{L}}$ is

$$\overline{\mathscr{L}} = \sum_{z=1}^{n-1}(n-z)\bigl[a(z)\log\alpha(z) - \alpha(z) + b(z)\log\beta(z) - \beta(z) + b(-z)\log\beta(-z) - \beta(-z)\bigr]. \tag{11}$$

Our approach involves maximizing this expression with respect to $\alpha(z)$ and $\beta(z)$ for given $a(z)$ and $b(z)$, which can be done using standard numerical methods. (Note that the expression separates into terms for the directed and undirected edges, so the two can be maximized independently.) The values of $a(z)$ and $b(z)$ in turn are calculated from Eqs. (3), (7), and (9), leading to an iterative method in which we first guess values for $\alpha(z)$ and $\beta(z)$, use them to calculate $q(R)$ and hence $a(z)$ and $b(z)$, then maximize $\overline{\mathscr{L}}$ to derive new values of α and β, and repeat to convergence. This is the classic expectation–maximization approach to model fitting.

Two further elements are needed to put this scheme into practice. First, we need to specify a parametrization for the functions α and β. We have found the results to be robust to the choice of parametrization, but in the results reported here we find α to be well represented by a Gaussian centered at the origin. The function β takes a more complicated form which we parametrize as a Fourier cosine series, keeping five terms and squaring to enforce nonnegativity, plus an additional Gaussian peak at the origin.

Second, the sum in the denominator of Eq. (3) is too large to be numerically tractable, so we approximate it using a Markov chain Monte Carlo method—we generate complete rankings R in proportion to the probability $q(R)$ given by Eq. (3) and average over them to calculate $a(z)$ and $b(z)$.

Results

We have applied the method of the previous section to the analysis of data from the US National Longitudinal Study of Adolescent Health (the "AddHealth" study), a large-scale multi-year study of social conditions for school students and young adults in the United States [10]. Using results from surveys conducted in 1994 and 1995, the study compiled friendship networks for over 90 000 students in schools covering US school grades 7 to 12 (ages 12 to 18 years). Schools were chosen to represent a broad range of socioeconomic conditions. High schools (grades 9 to 12) were paired with "feeder" middle schools (grades 7 and 8) so that networks spanning schools could be constructed.

To create the networks, each student was asked to select, from a list of students attending the same middle/high school combination, up to ten people with whom they were friends, with a maximum of five being male and

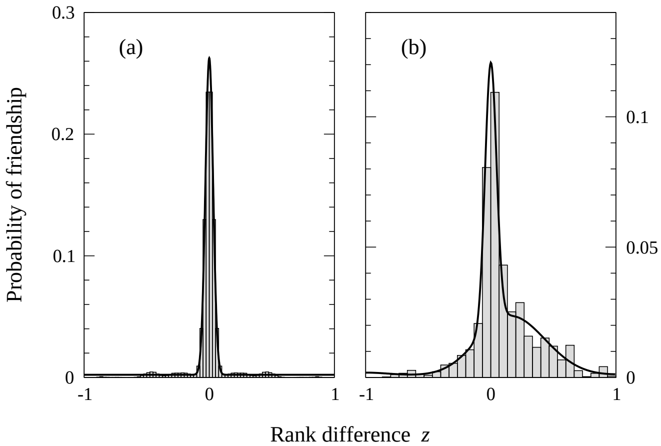

FIG. 1: (a) Probability of reciprocated friendships as a function of rank difference (normalized to run from -1 to 1). The histogram shows empirical results for a single example network; the solid curve is the fitted function $\alpha(z)$. (b) The equivalent plot for unreciprocated friendships.

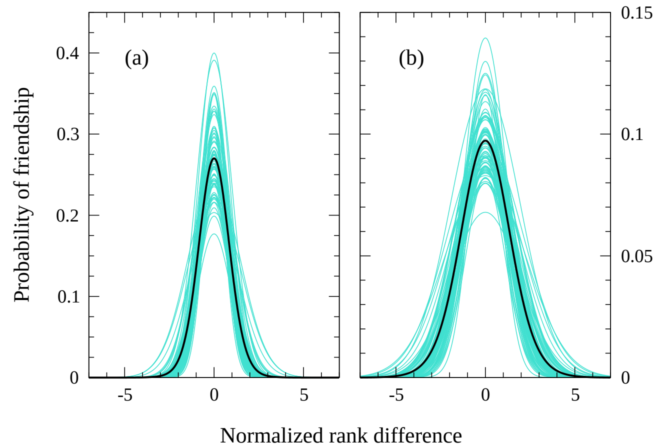

FIG. 2: The fitted central peak of the friendship probability distributions for (a) reciprocated and (b) unreciprocated friendships. The horizontal axes are measured in units of absolute (unrescaled) rank difference divided by average network degree. Each blue curve is a network. The bold black curves represent the mean.

five female. From these selections, 84 friendship networks were constructed ranging in size from tens to thousands of students, one for each middle/high school pair, along with accompanying data on the participants, including school grade, sex, and ethnicity. Some of the networks divide into more than one strongly connected component, in which case we restrict our analysis to the largest component only. We perform the EM analysis of the previous section on each network separately, repeating the iterative procedure until the rankings no longer change.

Figure 1 shows results for a typical network. In panel (a), the histogram shows the measured value of the quantity $a(z)$, Eq. (7), the empirical probability of a reciprocated friendship (technically the expected number of undirected edges) between a vertex pair with rank difference z, with the horizontal axis rescaled to run from -1 to 1 (rather than $-n$ to n). As the figure shows the probability is significantly different from zero only for small values of z, with a strong peak centered on the origin. The solid curve shows the fit of this peak by the Gaussian function $\alpha(z)$, which appears good. The fit is similarly good for most networks. The form of $a(z)$ tells us that most reciprocated friendships fall between individuals of closely similar rank: there is a good chance that two people with roughly equal rank will both claim the other as a friend, but very little chance that two people with very different ranks will do so. This result seems at first surprising, implying as it does that people must be able to determine their own and others' rank with high accuracy in order to form friendships, but a number of previous studies have suggested that indeed this is true [11].

Panel (b) of Fig. 1 shows $b(z)$, Eq. (9), for the same network, which is the probability of a directed edge between nodes with rank difference z. Again there is a strong central peak to the distribution, of width similar to that for the undirected edges, indicating that many unreciprocated friendships are between individuals of closely similar rank. However, the distribution also has a substantial asymmetric tail for positive values of the rank difference, indicating that in a significant fraction of cases individuals claim friendship with those ranked higher than themselves, but that those claims are not reciprocated. The black curve in the panel shows the best fit to the function $\beta(z)$ in the maximum-likelihood calculation.

The general forms of these distributions are similar across networks from different schools. They also show interesting scaling behavior. The widths of the central peaks for both undirected and directed edges, when measured in terms of raw (unrescaled) rank difference are, to a good approximation, simply proportional to the average degree of a vertex in the network. Figure 2 shows these peaks for 78 of the 84 networks on two plots, for undirected edges (panel (a)) and directed edges (panel (b)), rescaled by average degree, and the approximately constant width is clear. (The six networks not shown are all small enough that the central peaks for the directed edges can be fit by the other parameters of the model and thus a direct comparison is not appropriate.) This result indicates that individuals have, roughly speaking, a fixed probability of being friends with others close to them in rank, regardless of the size of the community as a whole—as the average number of friends increases, individuals look proportionately further afield in terms of rank to find their friends, but are no more likely to be friends with any particular individual of nearby rank.

Outside of the central peak, i.e., for friendships between individuals with markedly different ranks, there

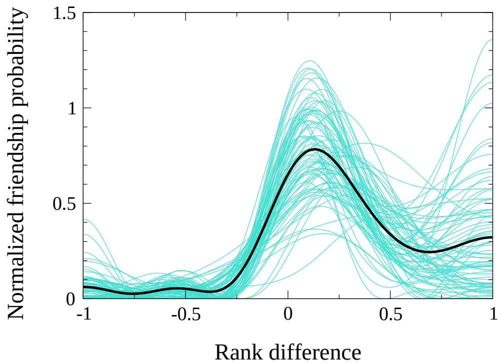

FIG. 3: The fitted probability function for unreciprocated friendships, minus its central peak. The horizontal axis measures rank difference rescaled to run from -1 to 1. Each blue curve is a network. The bold black curve is the mean.

are, to a good approximation, only unreciprocated friendships, and for these the shape of the probability distribution appears by contrast to be roughly constant when measured in terms of the rescaled rank of Fig. 1, which runs from -1 to 1. This probability, which is equal to the function $\beta(z)$ with the central Gaussian peak subtracted, is shown in Fig. 3 for the same 78 networks, rescaled vertically by the average probability of an edge to account for differing network sizes, and again the similarity of the functional form across networks is apparent, with low probability in the left half of the plot, indicating few claimed friendships with lower-ranked individuals, and higher probability on the right. The roughly constant shape suggests that, among the unreciprocated friendships, there is, for example, a roughly constant probability of the lowest-ranked student in the school claiming friendship with the highest-ranked, relative to other students, no matter how large the school may be.

The emerging picture of friendship patterns in these networks is one in which reciprocated friendships appear to fall almost entirely between individuals of closely similar rank. A significant fraction of the unreciprocated ones do the same, and moreover show similar scaling to their reciprocated counterparts, but the remainder seem to show a quite different behavior characterized by different scaling and by claims of friendship by lower-ranked individuals with substantially higher-ranked ones.

Discussion

Taking the results of the previous section as a whole, we conjecture that the rankings discovered by the analysis correlate, at least approximately, with social status. If we assume that reciprocated friendships—almost all of which fall in the central peak—correspond to friendships in the conventional sense of mutual interaction, then a further conjecture, on the basis of similar statistics, is that the unreciprocated friendships in the central peak are also mutual but, for one reason or another, only one side of the relationship is represented in the data. One explanation why one side might be missing is that respondents in the surveys were limited to listing only five male and five female friends, and so might not have been able to list all of their friendships.

On the other hand, one might conjecture that the unreciprocated claims of friendship with higher-ranked individuals, those in the tail of the distribution in Fig. 1b, correspond to "aspirational" friendships, hopes of friendship with higher-ranked individuals that are, at present at least, not returned. Note also how the tail falls off with increasing rank difference: individuals are more likely to claim friendship with others of only modestly higher rank, not vastly higher.

One way to test these conjectures is to look for correlations between the rankings and other characteristics of individuals in the networks. For instance, it is generally thought that social status is positively correlated with the number of people who claim you as a friend [9, 12]. Figure 4a tests this by plotting average rank over all individuals in all networks (averaged in the posterior distribution of Eq. (1)) as a function of network in-degree (the number of others who claim an individual as a friend). As the figure shows, there is a strong positive slope to the curve, with the most popular individuals being nearly twice as highly ranked on average as the least popular. Figure 4b shows the corresponding plot for out-degree, the number of individuals one claims as a friend, and here the connection is weaker, as one might expect—claiming many others as friends does not automatically confer high status upon an individual—although the correlation is still statistically significant. Figure 4c shows rank as a function of total degree, in-degree plus out-degree, which could be taken as a measure of total social activity, and here again the correlation is strong. For all three panels the correlations are significant, with p-values less than 0.001.

In addition to the network structure itself, we have additional data about each of the participants, including their age (school grade), sex, and ethnicity. The distributions of rank for each sex and for individual ethnicities turn out to be close to uniform—a member of either sex or any ethnic group is, to a good approximation, equally likely to receive any rank from 1 to n, indicating that there is essentially no effect of sex or ethnicity on rank. (A Kolmogorov–Smirnov test does reveal deviations from uniformity in some cases, but the deviations are small, with KS statistics $D < 0.08$ in all instances.) Age, however, is a different story. Figure 5 shows the rescaled rank of individuals in each grade from 7 to 12, averaged over all individuals in all networks, and here there is a clear correlation. Average rank increases by more than a factor of two from the youngest students to the oldest (a one-way ANOVA gives $p < 0.001$). Since older students are generally acknowledged to have higher social status [13], this result lends support to the identification of rank with status. A further interesting wrinkle can

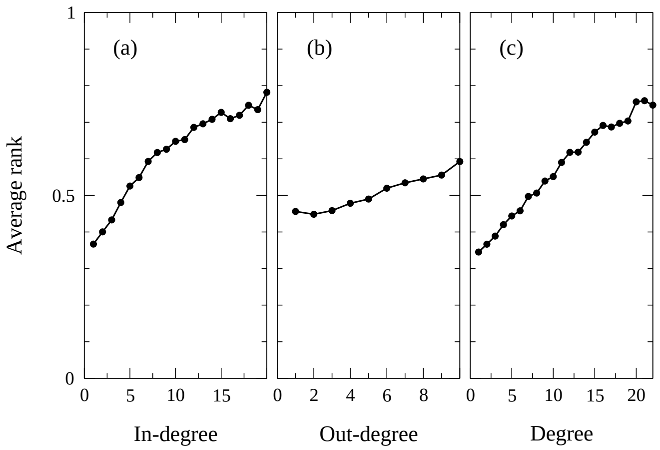

FIG. 4: Plots of rescaled rank versus degree, averaged over all individuals in all networks for (a) in-degree, (b) out-degree, and (c) the sum of degrees. Measurement errors are comparable with or smaller than the sizes of the data points and are not shown.

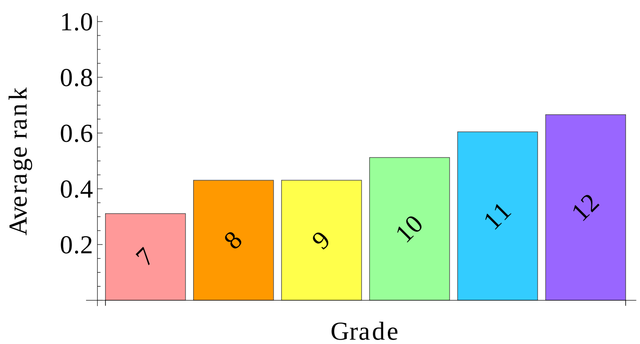

FIG. 5: Rescaled rank as a function of school grade, averaged over all individuals in all schools.

be seen in the results for the 8th and 9th grades. Unlike other pairs of consecutive grades, these two do not have a statistically significant difference in average rank (a t-test gives $p > 0.95$). This may reflect the fact that the 8th grade is the most senior grade in the feeder junior-high schools, before students move up to high school. When they are in the 8th grade, students are temporarily the oldest (and therefore highest status) students in school and hence may have a higher rank than would be expected were all students in a single school together.

Finally, in Fig. 6 we show an actual example of one of the networks, with nodes arranged vertically on a scale of inferred rank and colored according to grade. The increase of rank with grade is clearly visible, as is the fact that most undirected edges run between individuals of similar rank (and hence run horizontally in the figure).

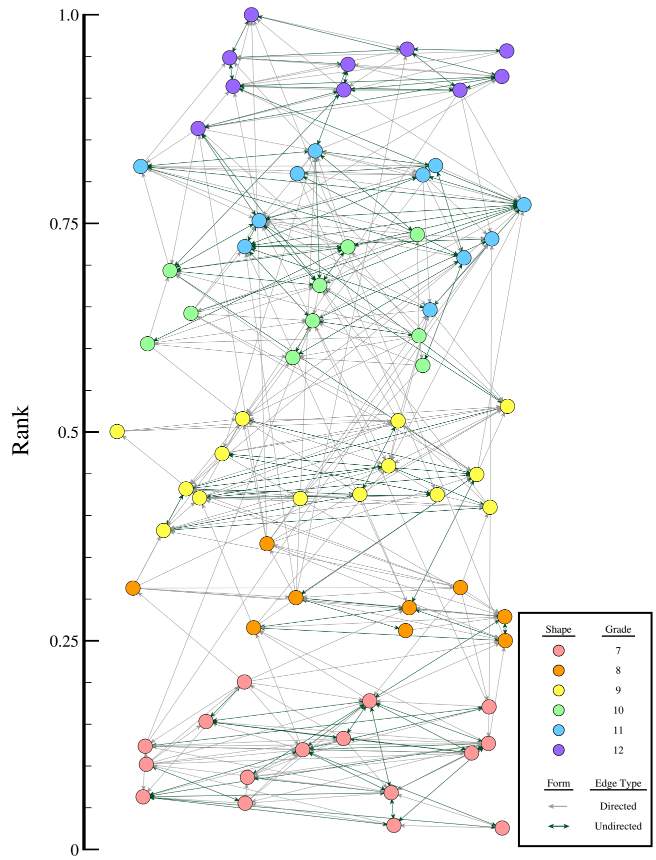

FIG. 6: A sample network with (rescaled) rank on the vertical axis, vertices colored according to grade, and undirected edges colored differently from directed edges. Rank is calculated as an average within the Monte Carlo calculation (i.e., an average over the posterior distribution of the model), rather than merely the maximum-likelihood ranking. Note the clear correlation between rank and grade in the network.

Conclusions

In this paper, we have analyzed a large set of networks of friendships between students in American high and junior-high schools, focusing particularly on the distinction between friendships claimed by both participating individuals and friendships claimed by only one individual. We find that students can be ranked from low to high such that most unreciprocated friendships consist of a lower-ranked individual claiming friendship with a higher-ranked one. We have developed a maximum-likelihood method for inferring such ranks from complete networks, taking both reciprocated and unreciprocated friendships into account, and we find that the rankings so derived correlate significantly with traditional measures of social status such as age and overall popularity, suggesting that the rankings may correspond to status. On the other hand, rankings seem to be essentially independent on average of other characteristics of the individuals involved such as sex or ethnicity.

There are a number of questions unanswered by our analysis. We have only limited data on the personal characteristics of participants. It would be interesting to test for correlation with other characteristics. Are rankings correlated, for instance, with academic achievement, number of siblings or birth order, number of Facebook friends, after-school activities, personality type, body mass index, wealth, or future career success? There is also the question of why a significant number of apparently close friendships are unreciprocated. One idea that has appeared in the literature is that some directed edges may correspond to new, temporary, or unstable friendships, which are either in the process of forming and will become reciprocated in the future, or will disappear over time [12, 14]. Evidence suggests that in practice about a half of the unreciprocated friendships do the former and a half the latter, and it is possible that the two behaviors correspond to the two classes of directed edges we identify in our analysis. A test of this hypothesis, however, would require time-resolved data—successive measurements of friendship patterns among the same group of individuals—data which at present we do not possess. Finally, there are potential applications of the statistical methods developed here to other directed networks in which direction might be correlated with ranking, such as networks of team or individual competition [15, 16] or dominance hierarchies in animal communities [17, 18].

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